

大物2



电磁学：研究场量的性质

1.1 电荷

1.2 库仑定律 $f = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$ Δ 点电荷 理想模型 Δ 符合电力叠加原理 $f = \sum f_i$

适用范围：微观 $\rightarrow$ 宏观 包括原子核

1.3 电场、电场强度

试探电荷 $E = \frac{F}{q}$ $f = qE$

1.4 点电荷电场及叠加原理

1. 点电荷的场强公式

$$f = \frac{Qq}{4\pi\epsilon_0 r^2} \hat{r}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

$\hat{r}$: 单位 $\hat{r}$

2. 场强叠加原理

$$E = \sum E_i = \sum \frac{q_i}{4\pi\epsilon_0 r_i^2} \hat{r}_i$$

$$E = \frac{1}{4\pi\epsilon_0 r^3} (-\vec{p} + 3(\vec{p} \cdot \hat{r}) \hat{r})$$

$r \gg l$ 电偶极子的场强



$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$= \frac{q}{4\pi\epsilon_0 r_+^2} \hat{r}_+ + \frac{q}{4\pi\epsilon_0 r_-^2} \hat{r}_-$$

$$= \frac{q}{4\pi\epsilon_0 r^2} \left(\frac{\vec{r}_+}{r_+} + \frac{\vec{r}_-}{r_-} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{\vec{r}_+}{r_+^3} - \frac{\vec{r}_-}{r_-^3} \right)$$

$$\text{电偶极子 } \vec{E} = \frac{1}{4\pi\epsilon_0 r^3} [-\vec{p} + 3(\vec{p} \cdot \hat{r}) \hat{r}]$$

$$\vec{p} = q\vec{l}$$

特殊情况

1. 逆线上

$$\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 r^3}$$



2. 垂直平分线上

$$\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 r^3}$$

3. 电荷连续分布的带电线

$$\lambda = \frac{dq}{dl}$$

$$\vec{E} = \int \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2} \hat{r}$$



$$dE_2 = dE \cos\theta$$

$$\cos\theta = \frac{z}{r}$$

$$E = E_2 = \int_{-L/2}^{L/2} \frac{\lambda dl}{4\pi\epsilon_0 r^2} \cos\theta$$

$$= \frac{\lambda z}{4\pi\epsilon_0 r^2} \int_{-L/2}^{L/2} \frac{dl}{r}$$

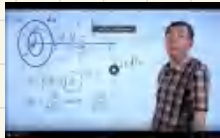
$$= \frac{\lambda z}{4\pi\epsilon_0 r^2} \cdot 2\pi R$$

$$= \frac{Qz}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}}$$

4. 电荷连续分布的带电面

$$E = \frac{dq}{ds} \hat{r}$$

$$\vec{E} = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}$$



$$dE_2 = \frac{dq}{4\pi\epsilon_0 r^2} \cos\theta$$

$$\cos\theta = \frac{z}{r}$$

$$= \frac{dq z}{4\pi\epsilon_0 r^3}$$

$$= \frac{\sigma r_2 dq dr_2 z}{4\pi\epsilon_0 (r_1^2 + z^2)^{3/2}}$$

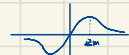
$$E = \int_{R_1}^{R_2} \int_0^{2\pi} \int_0^{2\pi} \frac{\sigma r_2 dq dr_2}{4\pi\epsilon_0 (r_1^2 + z^2)^{3/2}}$$

$$= \frac{\sigma}{4\pi\epsilon_0} \int_{R_1}^{R_2} \int_0^{2\pi} \frac{dr_2}{(r_1^2 + z^2)^{3/2}}$$

$$= \frac{\sigma}{2\epsilon_0} \int_{R_1}^{R_2} \frac{dr_2}{(r_1^2 + z^2)^{3/2}}$$

$$= \frac{\sigma}{2\epsilon_0} \left[\frac{r_2}{\sqrt{r_1^2 + z^2}} - \frac{r_1}{\sqrt{r_1^2 + z^2}} \right]$$

① E 的分布:



② $R_1 \rightarrow \infty, R_2 \rightarrow \infty$ 均匀带电无限大平面

$$E = |\vec{E}_2| = \frac{\sigma}{2\epsilon_0}$$

③ $R_1 \rightarrow \infty, R_2 = R$ 均匀带电圆面

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{1 + \frac{R^2}{z^2}}} \right]$$

球内距球心 r , 厚度 dr 球壳电荷:

$$dq = \rho dV = \rho \cdot 4\pi r^2 dr$$

$$= \frac{Q}{\frac{4}{3}\pi R^3} \cdot 4\pi r^2 dr$$

$\uparrow$

Δ 均匀带电球体 R, Q

$$E = \frac{Q}{4\pi\epsilon_0 R^2} \hat{r} \quad r < R \quad \text{球壳内 } E=0$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2} \hat{r} \quad r > R$$

均匀带电无限长直线: $E = \frac{\lambda}{2\pi\epsilon_0 r}$

均匀带电无限大平面: $E = \frac{\sigma}{2\epsilon_0}$

5. 电荷连续分布时带电体

$$\vec{E} = \int_{V_0} \frac{\rho dV}{4\pi\epsilon_0 r^2} \hat{r}$$



$$dV = r^2 \sin\theta dr d\theta d\phi$$



$$E_z = \int \frac{1}{4\pi\epsilon_0} \frac{(z-z')^2 \rho r^2 \sin\theta dr d\theta d\phi}{[r^2 + (z-z')^2]^{3/2}} \quad n = \begin{cases} \hat{z} & [A_0, A_0] \\ \hat{\phi} & [0, 2\pi] \\ \hat{\theta} & [0, \pi] \end{cases}$$

$$= \frac{\rho}{2\epsilon_0} \int_0^{\pi} \int_0^{2\pi} \left[\frac{(z+L)^2 + r^2}{2} - \sqrt{(z+L)^2 + r^2} - \sqrt{z^2 + r^2} + \sqrt{(z-L)^2 + r^2} \right] \sin\theta d\theta d\phi$$

$$L \rightarrow 0 \quad E_z = \frac{\rho}{2\epsilon_0} \left[\frac{1}{(z+L)^2} - \frac{1}{(z-L)^2} \right]$$

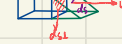
1.5 电通量

$$\text{面积 } dS = \vec{n} \cdot d\vec{s}$$

Δt 时间内流过 dS 流体的体积 $V_{\text{out}} dS$

$$\text{通量 } d\Phi = \vec{v} \cdot d\vec{s}$$

$$\text{电通量 } d\phi = \vec{E} \cdot d\vec{s}$$



闭合面元方向: 闭合面内指向面外

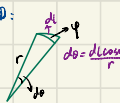
$$\phi = \oint_S \vec{E} \cdot d\vec{s}$$

通过任意封闭面的电通量 每一小面元视为匀强电场

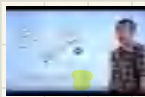
$$\phi = \oint_S \vec{E} \cdot d\vec{s} = \oint_S \vec{E} \cdot \vec{n} dS$$

立体角

平面:



立体:



闭合面所张立体角 $\left\{ \begin{array}{l} \text{对内} = 4\pi \\ \text{对外} = 0 \end{array} \right.$

$$d\Omega = \frac{\vec{r} \cdot d\vec{A}}{r^3}$$

静电场的高斯定理: 真空静电场中, V 面内电通量等于闭合面内电荷量 / ϵ_0

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{\sum q_{in}}{\epsilon_0}$$

闭合面内外电荷对 $\vec{E}$ 有贡献
但通量与面外无关

① 电荷在闭合面内

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r} \quad \oint_S \vec{E} \cdot d\vec{s} = \oint_S \vec{E} \cdot \hat{r} dS = \oint_S \frac{q}{4\pi\epsilon_0 r^2} dS = \frac{q}{4\pi\epsilon_0} \oint_S \frac{dS}{r^2} = \frac{q}{\epsilon_0}$$

② 外

$$\oint_S \frac{q}{4\pi\epsilon_0 r^2} d\Omega = 0$$

高斯定理应用 (电荷分布有对称性时 $\Rightarrow$ 电场强度 $\vec{E}$ 柱出来)

1. 均匀带电球面

$$\oint_S \vec{E} \cdot d\vec{s} = \oint_S E dS = E \oint_S dS = 4\pi R^2 E = \frac{\sum q_{in}}{\epsilon_0}$$

$$E = \frac{\sum q_{in}}{4\pi R^2 \epsilon_0}$$

球内: 高斯面内的 q

$$= \begin{cases} 0 & \text{电荷面外} \\ \frac{q}{4\pi R^2} & \text{球内} \end{cases}$$

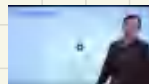
2. 无限长均匀带电圆柱



$$\oint_S \vec{E} \cdot d\vec{s} = \oint_S E dS = E \oint_S dS = E \cdot 2\pi r L = \frac{\sum q_{in}}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

3. 无限大均匀带电平板



$$E = \frac{\sigma}{2\epsilon_0}$$

$$\Delta \text{ 带电体面上 } \vec{E} = \frac{E_{in} + E_{out}}{2}$$

$\vec{E}_{表面} \pm \vec{E}_{外}$

$\left\{ \begin{array}{l} \text{内} \\ \text{外} \end{array} \right.$



电场线

静电场中电势积分总是与路径无关

环路定理

$$\oint_L \vec{E} \cdot d\vec{l} = 0$$

电势和叠加原理

$$\varphi_b - \varphi_a = \int_a^b \vec{E} \cdot d\vec{l}$$

点电荷电势

$$\varphi(b) - \varphi(a) = - \int_a^b \vec{E} \cdot d\vec{l} = - \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$

点电荷在有限远处电势为0

$$\varphi(r) = \int_r^\infty \vec{E} \cdot d\vec{l} = \frac{Q}{4\pi\epsilon_0} \frac{1}{r}$$

点电荷系 $\varphi = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$ (只数相加)

例: 连续电荷系

$$\vec{p} = q\vec{l}$$

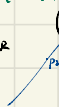


$$\varphi = \varphi_1 + \varphi_2 + \dots = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_1} + \frac{1}{r_2} + \dots \right) = \frac{Q}{4\pi\epsilon_0 r}$$

$r_1 \approx r - \frac{1}{2} \cos\theta$
 $r_2 \approx r + \frac{1}{2} \cos\theta$
 $r - r_1 \approx \frac{1}{2} \cos\theta$
 $r - r_2 \approx -\frac{1}{2} \cos\theta$

例: 球内/外电势 (均匀带电球壳)

$$E = \begin{cases} 0 & r < R \\ \frac{Q}{4\pi\epsilon_0 r^2} & r > R \end{cases}$$



$$\varphi_{p1} = W_{p1\infty} = \int_{p1}^\infty \vec{E} \cdot d\vec{l} = \int_{p1}^R 0 \cdot dl + \int_R^\infty \frac{Q}{4\pi\epsilon_0 r^2} \cdot dr = \frac{Q}{4\pi\epsilon_0 R}$$

$$\varphi_{p2} = \int_{p2}^\infty \frac{Q}{4\pi\epsilon_0 r^2} \cdot dr = \frac{Q}{4\pi\epsilon_0 r_2}$$

例: 平行板电容器和板间电势差

$$E = \frac{\sigma}{\epsilon_0}$$

$$\varphi = E \cdot d = \frac{\sigma d}{\epsilon_0}$$

电势梯度

$$\vec{E} = -\text{grad}\varphi = -\nabla\varphi = -\left(\frac{\partial\varphi}{\partial x} \hat{i} + \frac{\partial\varphi}{\partial y} \hat{j} + \frac{\partial\varphi}{\partial z} \hat{k} \right)$$

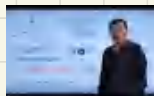
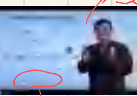
等势面

△ 电场处处垂直等势面

电势和静电场强

$$\vec{E} = -\nabla\varphi$$

点电荷间的相互作用能
没有无穷远处



作为整体在电场中能量
 $W = \frac{1}{2} \sum q_i \varphi_i$

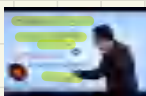
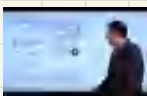
$$\vec{A} = \vec{p} \times \vec{E}$$

整体电势能

$$N \text{ 个电荷: } W = \frac{1}{2} \sum q_i \varphi_i$$

$$\varphi_i: N-1 \text{ 个电荷在 } q_i \text{ 处的电势}$$

$$= \frac{1}{2} \int \rho \varphi$$



0 或 r.

$$\frac{1}{2} \int \rho \frac{Q}{4\pi\epsilon_0 R} = \frac{\rho^2}{8\pi\epsilon_0 R}$$

介质中的电场

- 导体: 大量可自由移动电荷
- 绝缘体: 无
- 半导体: σ 介于两者间
- 超导体: 温度 $\downarrow$ 无电阻

导体

发生: all the time

导体静电平衡: 内部和表面的电荷定向移动

△ 等势体, 等势面

静电感应: (导体周围有电荷)

经典电动力学: 以不带电导体间不相互感应

导体: $E_{in}=0$ (宏观平均电场)

1. 内部处处不带电

2. 表面 $E_{in} = -\frac{\sigma}{\epsilon_0}$ 外法线方向



$$E_{in} dS = \frac{\sigma dS}{\epsilon_0}$$

尖端放电

3. 孤立带电导体表面电荷分布



正电荷大的电荷面密度大

有导体时静电场的计算

- 普通方法

1. 基本性质方程

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}, \quad \oint \vec{E} \cdot d\vec{l} = 0$$

2. 电荷守恒定律

3. 静电平衡条件 $E_{in}=0$ or $\phi=c$

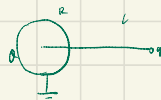
例: 无限大带电平面电场中, 平行放置一无限大金属平板, 求金属板两面电荷面密度

$$\text{电荷守恒: } q_1 + q_2 = 0 \\ \text{E 上不为力: } \frac{q_1 + q_2}{2\epsilon_0} = \frac{q_2}{\epsilon_0}$$



$$\frac{q_1}{4\pi\epsilon_0 a^2} + \frac{q_2}{4\pi\epsilon_0 a^2} + \frac{q_2}{4\pi\epsilon_0 a^2}$$

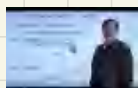
△ “接地”: 电势=0
≠ 不带电



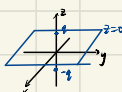
例

$$\frac{q}{4\pi\epsilon_0 a^2} + \frac{q}{4\pi\epsilon_0 a^2} = 0 \\ \frac{q}{\epsilon_0} = -\frac{q}{\epsilon_0} \quad Q = -\frac{q}{\epsilon_0}$$

二. 镜像法: △ 像电荷 - 且不能出现在所求电场空间区域



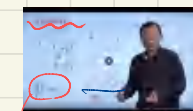
满足边界条件: $\phi=0 \Rightarrow q$ 和 $-q$ 所代替导体平板



边界条件: 边界 $\varphi=0$
 $\downarrow$
 找到像电荷 + 外面
 的电荷, 使得在原来导体表面
 上 $\varphi=0$



$$q' = -q, \quad q' = -q$$



$$q' = -q = -\frac{q}{2}$$

导体与静电屏蔽



由高斯定理, 腔内无净电荷, 腔内无电场, 总是静电屏蔽, 与腔外有无电荷无关

导体内电场为 0
 $\oint \vec{E} \cdot d\vec{s} = 0$



电容与电容器

一 孤立导体的电容 $C = \frac{Q}{\varphi}$

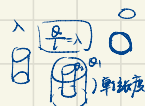
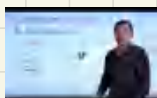
$$\varphi = \frac{Q}{4\pi\epsilon_0 R}, \quad C = 4\pi\epsilon_0 R$$

二 导体组的电容



$$\text{平行板电容: } U = Ed = \frac{Q}{\epsilon_0} d = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{U} = \frac{\epsilon_0 A}{d}$$



自由电荷
 极化电荷
 极化发生在电介质内/表面

能量储存在场中

三 电容器的储能

$$W = \frac{1}{2} Q \Delta\varphi = \frac{1}{2} \int \epsilon_0 \vec{E} \cdot d\vec{s} = \frac{1}{2} \epsilon_0 \int E^2 dV$$

电场能量密度为 $W_0 = \frac{1}{2} \epsilon_0 E^2$

$$= \frac{1}{2} \int \epsilon_0 E_x^2 dV + \frac{1}{2} \int \epsilon_0 E_y^2 dV$$

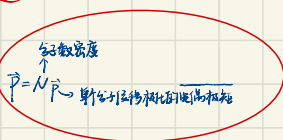
$$= \frac{1}{2} \int \epsilon_0 E_x^2 dV + \frac{1}{2} \int \epsilon_0 E_y^2 dV$$

$$= \frac{1}{2} Q \Delta\varphi = \frac{1}{2} C (\Delta\varphi)^2$$

电介质对电场的影响 (实际上是通过对极化电荷作用来)

$C = C_0 \epsilon_r$, 一般 $\epsilon_r > 1$, 电介质的相对介电常数

极化强度: 单位体积的总电偶极矩



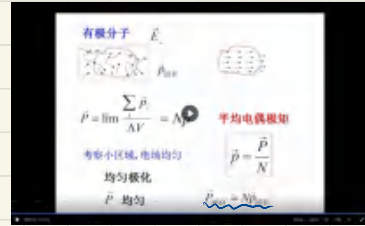
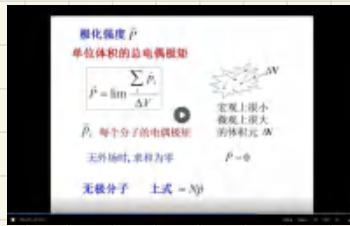
ϵ_r 是 ϵ 比例

$$C = \epsilon_r \epsilon_0 \frac{A}{d}$$

无极分子组成的电介质在 E 诱导下偶极矩

↑ 定向极化

无极分子: 无外场时, 热运动, 无规取向, 单位体积总偶极矩处处为 0
 有外场: 取向极化
 定向排列 ($\rightarrow$ 为 E 方向的更多)



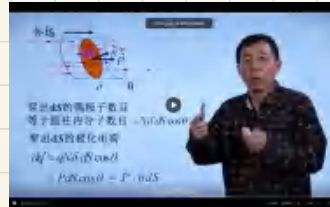
极化电荷与极化强度关系

$P: \odot \rightarrow \ominus$

$$q' = -\oint P \cdot dS$$

$$\vec{P} = (\epsilon_r - 1)\epsilon_0 \vec{E}$$

极化: 出现在电介质内(极化电荷)



$$q' = \vec{P} \cdot \vec{n}$$

电介质表面
极化电荷面密度

$$\sigma' = P_n = \vec{P} \cdot \vec{n}$$

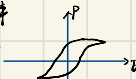
电介质的极化规律

1. 各向同性线性电介质 $\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$
自由电荷 + 极化电荷 create

2. 各向异性线性电介质

3. 铁电体

1. 滞回现象



2. 介电常数大 $\epsilon_r \sim 10^2 \sim 10^4$

3. 居里点 T_c

T 以下: 铁电体

T 以上: 电介质

4. 压电现象

有电介质时静电场计算

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

$$q' = -\oint P \cdot dS \quad \text{or 均匀极化时 } \sigma' = \vec{P} \cdot \vec{n}$$

$$\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$$

当各侧同性均匀电介质充满电场空间 $\epsilon = \frac{\epsilon_0}{\epsilon_r}$ ϵ_0 : 真空情况

电位移量 $\vec{D}$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}, \vec{P} = \epsilon_0 (\epsilon_r - 1) \vec{E} = \epsilon_0 \epsilon_r \vec{E}$$

C/m^2 量纲

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0} = \frac{\sum q_{\text{自}} + \sum q_{\text{极}}}{\epsilon_0}$$

电介质绝缘无自由电荷

∴ 在同一介质内使用高斯定理 $\oint_S \vec{D} \cdot d\vec{s} = \sum q_{\text{自}}$

$$\oint_S \vec{D} \cdot d\vec{s} = \sum q_{\text{自}}$$

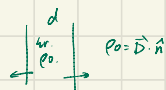
某种对称性 $\Rightarrow \vec{D} \propto \vec{E} \propto \vec{P} \propto \rho \propto q_{\text{自}}$

$$\vec{P} = (1 - \frac{1}{\epsilon_r}) \vec{D}$$

$$\vec{P}' = -(1 - \frac{1}{\epsilon_r}) \rho_0$$

ρ_0 : 自由电荷密度

ρ' : 极化电荷密度



导体球壳上的均匀电荷分布 (同心) 如图示, 导体带电 Q

求: 1) 场强分布
2) 紧贴导体球表面处的极化电荷
3) 两介质交界处的极化电荷

解: 根据球对称性 $\oint \vec{D} \cdot d\vec{s} = Q_{\text{自}}$

$$\vec{D} = \begin{cases} 0 & r < R \\ \frac{Q}{4\pi r^2} \hat{r} & r > R \end{cases}$$

1) 场的分布

$$\vec{E} = \frac{\vec{D}}{\epsilon_0 \epsilon_r}$$

$$\vec{P} = \left(1 - \frac{1}{\epsilon_r}\right) \vec{D}$$

极化区域别有 $E_{\text{自}} = 0, E' = 0$

$$E_{\text{自}} = \frac{Q}{4\pi \epsilon_0 \epsilon_r r^2} \hat{r}, E' = \left(1 - \frac{1}{\epsilon_r}\right) \frac{Q}{4\pi r^2} \hat{r}$$

电介质极化, 不是均匀极化

极化电荷密度?

$$q' = -\oint \vec{P} \cdot d\vec{s} = -\oint \vec{P} \cdot \hat{n} \cdot d\vec{s}$$

$$\vec{P} = \left(1 - \frac{1}{\epsilon_r}\right) \vec{D}$$

自由电荷密度为零处
极化电荷密度亦为零

注意: 点电荷 Q 周围极化面电荷!

2) 求紧贴导体球表面处的极化电荷

$$\sigma' = \vec{P} \cdot \hat{n} \Big|_{r=R}$$

$$= P_{\text{自}} \Big|_{r=R}$$

$$= \frac{Q}{4\pi R^2} \left(1 - \frac{1}{\epsilon_r}\right)$$

3) 两介质交界面极化电荷

$$\sigma'_{\text{交界}} = (P_1 - P_2)_{\text{交界}} = \frac{Q}{4\pi R^2} \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2}\right)$$

一无限大各侧同性均匀介质的平板, 厚度为 d , 相对介电常数为 ϵ_r , 内部均匀分布体电荷密度为 ρ_0 的自由电荷

求: 介质板内, 场强 E, D, P

解: 选取轴对称, 取高斯面 S

(1) E, P 垂直板面
(2) x 轴处 $E = 0$
(3) 两侧对称

以 $x=0$ 处的面为对称

$$\vec{E} = \frac{\vec{D}}{\epsilon_0 \epsilon_r}$$

作正圆形高斯面 S_1

$$E_{\text{自}} = \frac{Q}{2\epsilon_0 \epsilon_r} = \frac{\rho_0 d}{2\epsilon_0 \epsilon_r}$$

或取圆柱 S_2

$$E_{\text{自}} = \frac{Q}{2\epsilon_0 \epsilon_r} = \frac{\rho_0 d}{2\epsilon_0 \epsilon_r}$$

求: 介质板内, 场强 E, D, P

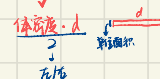
解: 选取轴对称, 取高斯面 S

(1) E, P 垂直板面
(2) x 轴处 $E = 0$
(3) 两侧对称

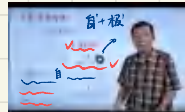
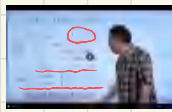
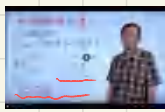
极化电荷密度?

$$\vec{P} = \left(1 - \frac{1}{\epsilon_r}\right) \vec{D}$$

极化电荷密度?

$$\sigma' = P_{\text{自}} \Big|_{x=d/2} = \frac{Q}{4\pi R^2} \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2}\right)$$


边值关系



与外界处
 D_n 一样

上面 $\cap$ 只小相同
 下面 $\cup$ 只小相同

$$\frac{D_1}{\epsilon_1} = \frac{D_2}{\epsilon_2} \quad E = \frac{q'_{12}}{4\pi\epsilon_0 r^2} \hat{r}$$

$$= \frac{q}{4\pi r^2} \frac{1}{\epsilon_1 + \epsilon_2}$$

$$\Rightarrow q'_{12}$$

均匀时,

$$D_n = D_n = \epsilon_0 \epsilon_r E$$

$$D_{\text{总}} = \epsilon_0 (\epsilon_r - 1) E$$

$$D_{\text{总}} = D_0 + D_{\text{极}} = \epsilon_0 E$$

$$\frac{D_0 + D_{\text{极}}}{\epsilon_0} = \frac{q}{4\pi r^2} \quad \text{成比例}$$

有介质时静电场能量

一、有介质时的电容器电容

(均匀时)

$$\left| \begin{array}{l} \epsilon_r \\ d \end{array} \right| \begin{array}{l} S \\ \end{array} \quad \begin{array}{l} E = \frac{q}{\epsilon_r} \\ C = \frac{Q}{V} \end{array} \quad V = \frac{q}{\epsilon_r} \quad \epsilon_0 = \frac{q_0}{\epsilon_0} \quad \vec{D} = \epsilon_0 \epsilon_r E$$

$$C = \frac{Q}{V} \quad C = \epsilon_0 \epsilon_r \frac{S}{d} \quad \begin{array}{l} = \epsilon_0 \epsilon_r \\ = \epsilon_0 \end{array}$$

$\vec{D}$ 等于平均板上自由电荷密度

二、场强密度 有介质, 电容器电容公式相同

$$W_E = \frac{Q^2}{2C} = \frac{1}{2} QV = \frac{1}{2} CV^2$$

$$= \frac{1}{2} QV = \frac{1}{2} Q \cdot \frac{Q}{C} = \frac{1}{2} DE \cdot V$$

$$w_E = \frac{W_E}{V} = \frac{1}{2} DE = \frac{1}{2} \vec{D} \cdot \vec{E}$$

电场能量密度 $w_E = \frac{1}{2} \vec{D} \cdot \vec{E}$

$$\text{电流密度 } j = \frac{dI}{dS_{\perp}}$$

$$dq = |v| dt dS_{\perp} n_q$$

$$dI = |v| dS_{\perp} n_q = \vec{v} \cdot d\vec{s} n_q = j \cdot d\vec{s}$$

$$\vec{j} = \frac{dI}{dS_{\perp}} = \vec{v} n_q$$

$$I = \oint \vec{j} \cdot d\vec{s}$$

电流连续性方程

若导体回路中电流线闭合,
 则电荷分布固定 $\Rightarrow$ 稳恒电流

$$\oint \vec{j} \cdot d\vec{s} = - \frac{dq_{\text{总}}}{dt}$$

电流线发生于正电荷减少地方, 终止于正电荷增加地方

稳恒电流: 电流场中每一点电流密度大小, 方向不随时间改变

$$\oint \vec{j} \cdot d\vec{s} = 0$$

$\downarrow$

① 稳恒电流电路必须闭合

② 导体表面电流密度矢量无法向分量

基尔霍夫第一定律: 电路任一节点处, 流入电流强度之和 = 流出节点电流强度之和

稳恒电场: 电荷上静电场

导体处于静电平衡: 稳恒电流的电场伴随电荷定向移动, 导体内部静电场不为0

电动势: 把单位正电荷从电源内部由负极移向正极过程, 非静电力作的功

接成闭合电路做功 = 0
 ↳ 环路定理 闭合电路

$$\oint \vec{E} \cdot d\vec{l} = \oint_L \vec{E} \cdot d\vec{l}$$

温差电动势成因:

- ① 同种金属中, 温差 $\rightarrow$ 自由电子热扩散
- ② 不同金属中, 自由电子浓度不同, 接触产生与温度有关扩散

欧姆定律  $R = \rho \frac{L}{S}$ ρ : 电阻率

$$I = \frac{U}{R}, U = E L, I = j S$$

$$R = \frac{E L}{j S} = \rho \frac{L}{S}$$

$$\therefore E = \rho j = \frac{j}{\sigma} \quad \sigma \text{ 电导率} \quad \sigma = \frac{1}{\rho}$$

$$\vec{E} = \frac{j}{\sigma} \quad j = \sigma \vec{E} \quad (\text{无电场时})$$

电流是电场驱动下电荷定向移动

对于稳恒电流, $\oint j \cdot d\vec{s} = 0 \quad \therefore \oint \vec{E} \cdot d\vec{s} = 0 \quad \therefore$ 均匀导体内处处无电荷

稳恒电流导线, 且导体材料各处均匀, 电荷分布在表面或界面 

焦耳定律

$$P = UI = E L \cdot j S$$

$$= \frac{j^2}{\sigma} V$$

$$\therefore P = \frac{j^2}{\sigma} \quad \text{单位V电功率}$$

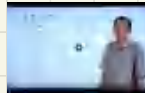
有电阻时, $j = \sigma(\vec{E} + \vec{E}_k)$ 欧姆定律微分形式



$$\oint_L \vec{E} \cdot d\vec{l} = \oint_L (\frac{j}{\sigma} - \vec{E}_k) \cdot d\vec{l} = 0$$

$$\sum (I_i) R_i + \sum (I_i) \omega_i = 0$$

$$I_i \text{ 与 } L \text{ 反向一致时} \quad \omega_i = \frac{1}{\sigma} \frac{dI_i}{dt} \quad \frac{1}{\sigma} \frac{dI_i}{dt} = \omega_i$$



叠加原理, 流入 I_1 , 流出 I_2
 合: $\frac{I_1}{\sigma}$

求B: 以为错误的情况而
11

$$\textcircled{1} \oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

if B是可以提出来的

求出来的也是整个闭合工区内的

$$\textcircled{2} d\vec{B} = \frac{\mu_0 I d\vec{l} \times \vec{r}}{4\pi r^2} \quad \text{用于积分}$$

$$\textcircled{3} \oint_S \vec{B} \cdot d\vec{s} = 0$$

④  $B \cdot 2\pi r = \mu_0 I$
 $\therefore B = \frac{\mu_0 I}{2\pi r}$

⑤ 

$$B = \frac{\mu_0 I}{2} \cdot \frac{R^2}{(R^2 + x^2)^{\frac{3}{2}}}$$

$$B_0 = \frac{\mu_0 I}{2R}$$

②

$dy = \frac{h \sin \theta \cdot d\theta}{4\pi h}$

$l = \frac{h}{\sin \theta} \quad dl = \frac{h}{\sin^2 \theta} d\theta$

$$B = \int_0^{\theta_2} \frac{\mu_0 I}{4\pi h} \frac{dl \cdot \sin \theta}{h^2} \sin^2 \theta$$
$$= \int_0^{\theta_2} \frac{\mu_0 I}{4\pi h^2} \sin^2 \theta \cdot \frac{h}{\sin^2 \theta} d\theta$$
$$= \int_0^{\theta_2} \frac{\mu_0 I}{4\pi h} \sin \theta d\theta$$
$$= \frac{\mu_0 I}{4\pi h} (\cos \theta_1 - \cos \theta_2)$$

$\frac{d\theta}{d\theta}$

电流磁效应

磁场: 对运动电荷有力 $\vec{f} = q\vec{v} \times \vec{B} + q\vec{E}$

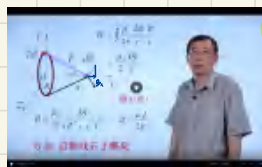
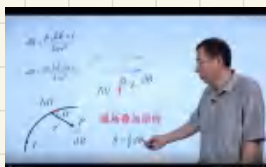
磁通密度/磁感应强度 B $1T=10^4G$

毕-萨-拉定律

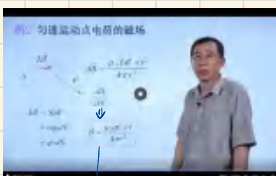


$$dB = \frac{\mu_0 I dl \times \hat{r}}{4\pi r^2}$$

$$\mu_0: \text{真空中磁导率 } \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

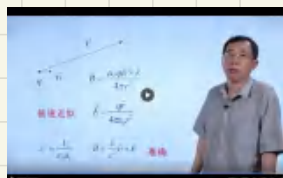


圓心處: $B = \frac{\mu_0 I}{2R}$



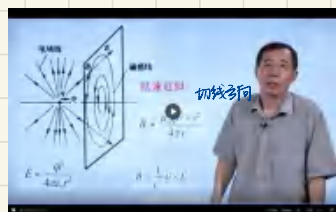
单个载流子产生的磁场

$$\vec{B} = \frac{\mu_0 q \vec{v} \times \vec{r}}{4\pi r^2}$$



$$\vec{B} = \frac{1}{c^2} \vec{v} \times \vec{E}$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$



载流线圈的磁矩及其磁场



$$\vec{p}_m = I \vec{S}$$

在远处产生的磁场 $\vec{B} = \frac{\mu_0}{4\pi r^3} [-\vec{p}_m + 3(\vec{p}_m \cdot \hat{r})\hat{r}]$

vs. 电荷极子在远处产生的电场 $\vec{E} = \frac{1}{4\pi\epsilon_0 r^3} [-\vec{p}_e + 3(\vec{p}_e \cdot \hat{r})\hat{r}]$

磁场高斯定律

一个电流元对闭合曲面磁通量为0

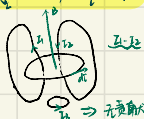
$$\phi_m = \oint_S \vec{B} \cdot d\vec{s} \quad \text{w.} \quad \oint_S \vec{B} \cdot d\vec{s} = q_m \quad \text{磁荷}$$

安培环路定理及应用

磁场是轴矢量,不能有
平行于镜面对称面的分量

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{in}$$

恒定磁场中, B沿任一闭合环路的线积分, 等于穿过该环路所有电流的代数和的 μ_0 倍



I_1, I_2 : 真子集 } 共同产生 B
 I_3 : 不真子集

十: 电流方向与右手螺旋方向一致

闭

$$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$



管内均匀场 $B = \mu_0 n I$
管外 $B = 0$



$$2BL = \mu_0 I \quad B = \frac{\mu_0 I}{2L}$$

$$n = \frac{N}{L} \text{ 单位长度匝数}$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = \vec{A} \cdot (\vec{B} \times \vec{C})$$

多个电流回路 同轴线圈积分

$$\oint \vec{B} \cdot d\vec{l} = \oint \sum \vec{B}_i \cdot d\vec{l} = \mu_0 \sum I_i \quad (I_i \text{ 为总})$$

$$\oint \vec{B}_i \cdot d\vec{l} = \begin{cases} \pm \mu_0 I_i & \text{穿过回路} \\ 0 & \text{不穿过回路} \end{cases}$$

特别：右手螺旋规定

带电粒子在磁场中的运动

$$\text{洛伦兹力 } \vec{F}_m = q \vec{v} \times \vec{B}$$

速度方向与磁场方向垂直

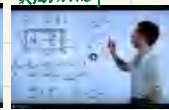
$$\vec{v} \perp \vec{B} \quad qvB = m \frac{v^2}{R} \quad R = \frac{mv}{qB} \quad T = \frac{2\pi m}{qB}$$

$\vec{v}$ 与 $\vec{B}$ 不平行不垂直



洛伦兹力: $B \perp \vec{v}$ 时
但 $W_{洛} = 0$, $E_{洛} = 0$

霍尔效应



$$qE = qvB = q \frac{V}{h}$$

$$j = nVq$$

$$V_H = R_H \frac{IB}{nq} \quad R_H = \frac{1}{nq}$$

$$R_H = \frac{B}{nqB}$$

安培力



闭合导线在B中受力为0



电阻率 $\rho = \frac{m}{nq} \cdot \frac{1}{\tau} = \frac{m}{nq} \left(\frac{1}{\tau_{e0}} + \frac{1}{\tau_{e1}} + \dots \right)$

τ : 平均2次碰撞之间的时间

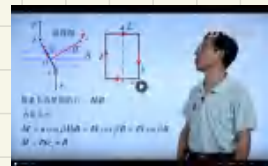
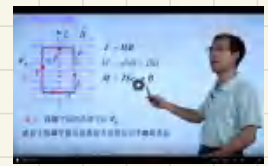
$\frac{1}{\tau}$: 碰撞频率

似稳电路: 特征时间 $\tau \gg \frac{1}{\omega}$

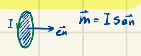
交流电似稳近似: $\frac{1}{\omega} \gg \frac{1}{\tau}$



载流线圈在均匀磁场中受的磁力矩 $\vec{M} = SI \vec{e}_n \times \vec{B} = \vec{m} \times \vec{B}$ 任意形状



磁矩: $\vec{m} = I S \vec{e}_n$ (指的是小载流线圈, 磁场均匀)



电流磁矩 $\vec{p}$

磁矩 $\vec{m}$

$\vec{p} = q \vec{r}$

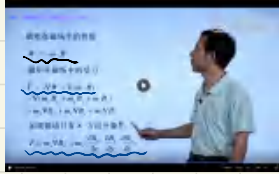
$\vec{m} = \vec{m} \times \vec{B}$

$W = -\vec{p} \cdot \vec{E}$

$W = -\vec{m} \cdot \vec{B}$



$W_m = -\vec{m} \cdot \vec{B}$



$\vec{F} = -\nabla W$
 $= \nabla(\vec{m} \cdot \vec{B})$

★ 磁矩在磁场中受力

$F = m \nabla B_x$
 $= m \left(\frac{\partial B_x}{\partial x}, \frac{\partial B_x}{\partial y}, \frac{\partial B_x}{\partial z} \right)$

磁场中的磁介质

$\vec{B} = \mu_r \vec{B}_0$

一般抗磁性: $\mu_r < 1$ 且与1接近

顺磁性: $\mu_r > 1$ 且与1接近

铁磁性: $\mu_r \gg 1$

原子的磁矩



轨道磁矩 $\vec{m}_L = -\frac{e}{2m_e} \vec{L}$
自旋磁矩 $\vec{m}_S = -\frac{e}{m_e} \vec{S}$



电子: $\vec{m}_e = -g \frac{e}{2m_e} \vec{S}$
总磁矩

磁介质的磁化

顺磁性物质的磁化

磁化方向与外磁场方向一致

$$M = \lim_{\Delta V \rightarrow 0} \frac{\sum \vec{m}}{\Delta V} = \frac{1}{V} \sum \vec{m} = n \vec{m} \cos \theta$$

单位体积分子数

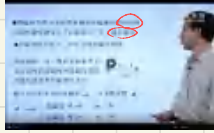


抗磁性物质的磁化



方向相反

总结



⇒ 叠加

- 抗磁性 $B = \mu_0 B_0 < B_0$
- 顺磁性 $B = \mu_0 B_0 > B_0$

$$\vec{M} = \frac{\mu_0}{4\pi} \nabla \times \vec{B}$$

$$= n \vec{m}$$

⇒ 磁化电流 感生磁场

$$\text{磁化电流密度 } \vec{j}' = \frac{d\vec{M}}{dl}$$

作业练习：表面产生磁化电流
不：第1内



$$I' = \oint \vec{j}' \cdot d\vec{l}$$



$$j' = \frac{dM}{dl} = M \cdot \frac{1}{l}$$

$$d\vec{l} = M + dl$$

$$\vec{j}' = \vec{M} \times \vec{n}$$



$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\oint \vec{H} \cdot d\vec{l} = \sum I_{\text{自由}} + I_{\text{束缚}}$$

R有自由电流

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (\sum I_{\text{自由}} + \sum I_{\text{束缚}})$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{M} = \frac{\mu_0 - 1}{\mu_0} \vec{B}$$

$$I_{\text{总}} = \oint \frac{\vec{B}}{\mu_0} \cdot d\vec{l}$$

$$= \oint \frac{\mu_0}{4\pi} \nabla \times \vec{B} \cdot d\vec{l}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu_r \vec{H}$$

$$\vec{M} = (\mu_r - 1) \vec{H}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{M} = \frac{\mu_r - 1}{\mu_0} \vec{B}$$



磁场界面关系

$$\oint \vec{B} \cdot d\vec{l} = B_1 S_1 - B_2 S_1 = 0 \Rightarrow B_1 = B_2$$

$$H_1 = H_2$$

界面两侧

磁感应强度法向分量连续, $B_{1n} = B_{2n}$
 磁场强度切向分量相等, $H_{1t} = H_{2t}$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

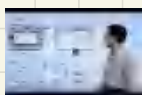
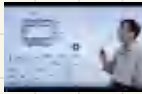
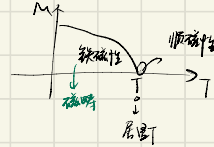
$$\therefore \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2} \Rightarrow \mu_1 = \mu_2 \mu_r$$

$$\tan \theta_1 = \frac{B_{1t}}{B_{1n}} = \frac{\mu_1 H_{1t}}{\mu_1 H_{1n}} = \frac{\mu_1}{\mu_2} \tan \theta_2$$

$$\tan \theta_1 = \frac{\mu_2}{\mu_1} \tan \theta_2 \Rightarrow \mu \uparrow \Rightarrow \theta \downarrow \Rightarrow \text{静磁屏蔽}$$



磁路: 磁感线集中通路 Δ 磁路中 $B \cdot S$ 处处相等



$$\Phi_m = NI \quad \text{磁通量}$$

$$R = \frac{l}{\mu S} \quad \text{磁阻}$$

$$\mu = \mu_0 \mu_r$$

铁磁性材料



去磁过程



硬磁 $\Rightarrow$ 磁滞损耗大
 软磁 $\Rightarrow$ 小

Δ 磁畴各向异性
 易轴: 易磁化
 难轴: 难磁化



$$\epsilon = \frac{dq}{dt}$$

任意磁场通过封闭曲面 $\Phi = 0 \Rightarrow$ 磁通连续定理

$$\oint \vec{v} \times \vec{B} \cdot d\vec{l} = \frac{1}{\epsilon} \oint \vec{v} \cdot d\vec{q} \Rightarrow \oint \vec{v} \times \vec{B} \cdot d\vec{l} = \frac{1}{\epsilon} \oint \vec{v} \cdot d\vec{q}$$

$$\oint \vec{v} \times \vec{B} \cdot d\vec{l} = \frac{1}{\epsilon} \oint \vec{v} \cdot d\vec{q}$$



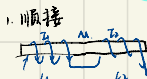
$$\oint \vec{v} \times \vec{B} \cdot d\vec{l} = - \frac{1}{\epsilon} \oint \vec{v} \cdot d\vec{q}$$



自感

$$\Phi_m = -L \frac{dI}{dt}$$

$$L = \frac{\Phi}{I} = \frac{N \Phi}{I}$$



$$L = L_1 + L_2 + 2M$$

$$L = L_1 + L_2 - 2M$$

$$\frac{1}{2} L I^2 > 0 \Rightarrow M < \sqrt{L_1 L_2}$$



$$L = \mu_0 \mu_r N^2 V$$

★ η^2

互感电动势的符号

$$\text{互感电动势 } \mathcal{E} = -\frac{d\Phi}{dt} = -M \frac{dI}{dt}$$

$$M_{21} = \frac{\Phi_{21}}{I_1} = \frac{N_2 \Phi_1}{I_1}$$

$$M \leq \sqrt{L_1 L_2}$$

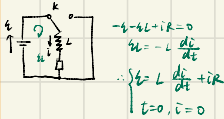
只要没漏磁 除非重叠



$$\frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 \pm M I^2$$

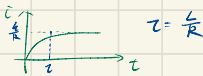
RL电路暂态过程

初态 → 稳态

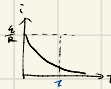


$$\begin{aligned} -E - L \frac{dI}{dt} + IR &= 0 \\ L \frac{dI}{dt} + IR &= E \end{aligned}$$

$$I = \frac{E}{R} (1 - e^{-\frac{R}{L}t})$$



$$\begin{aligned} L \frac{dI}{dt} + IR &= 0 \\ t=0, I &= \frac{E}{R} \\ I = \frac{E}{R} e^{-\frac{R}{L}t} = \frac{E}{R} e^{-\frac{t}{\tau}} \end{aligned}$$



通有电流 I 的自感线圈 L 中的磁能 $W_L = \frac{1}{2} L I^2$

对于长直螺线管:

$$L = \mu_0 \mu_r N^2 V, B = \mu_0 \mu_r n I, H = \frac{B}{\mu_0 \mu_r}$$

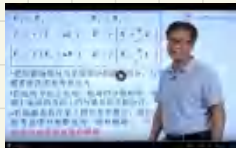
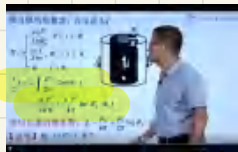
$$W_m = \frac{1}{2} L I^2 = \frac{1}{2} B H V$$

$$\text{磁场能量密度: } W_m = \frac{1}{2} B H = \frac{B^2}{2 \mu_0 \mu_r}$$

电磁场能量密度 $w = \frac{1}{2} (D \cdot E + B \cdot H)$

通过磁场求自感

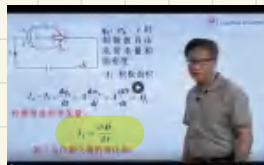
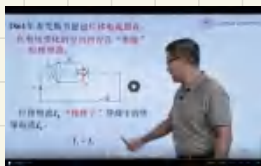
$$\frac{1}{2} L I^2 = \frac{1}{2} \iiint \vec{B} \cdot \vec{H} dV = \frac{1}{2} \iiint \frac{B^2}{\mu_0 \mu_r} dV$$



密度
↓
计算表达式

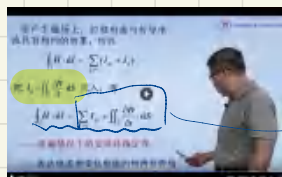
麦克斯韦方程组和电磁场

位移电流假设 变化的电场 $\Rightarrow$ 产生磁场



$$\Delta \vec{j} = \frac{\partial \rho}{\partial t} = \frac{I}{S}$$

$$I = \iint_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s} = \oint \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s}$$



$$\star \vec{j}_0 = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$I_d = \iint_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

穿过 0

$$\oint \vec{H} \cdot d\vec{l} = \sum I_i + \iint_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

$$\iint (\vec{j} + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{s}$$



方程组

$$\oint_S \vec{D} \cdot d\vec{s} = \iint_V \rho_0 dv = \iint_V \rho_0 dv$$

$$\oint_L \vec{E} \cdot d\vec{l} = - \iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\oint_L \vec{H} \cdot d\vec{l} = \iint_S (\vec{j}_0 + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{s}$$

$$\nabla \cdot \vec{D} = \rho_0$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t}$$

$$- \frac{\partial \Phi}{\partial t} = \oint_L \vec{E} \cdot d\vec{l}$$

$$\nabla = \vec{e} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$$

边界关系

$$D_{1n} = D_{2n} \text{ (界面上无自由电荷)}$$

$$E_{1t} = E_{2t}$$

$$B_{1n} = B_{2n}$$

$$H_{1t} = H_{2t} \text{ (界面上无传导电流)}$$

介质的性质

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0 \mu_r}$$

$$\vec{j} = \sigma (\vec{E} + \vec{E}) \quad \kappa \text{ 非静电力}$$

洛伦兹力公式

$$\vec{F} = q \vec{E} + q \vec{v} \times \vec{B}$$

电磁波

$$\epsilon = \epsilon_0 \epsilon_r$$

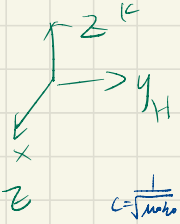
$$\mu = \mu_0 \mu_r$$

$$H = \frac{B_0}{\mu_0} - M$$

$$B_0 = \mu_0 (H + M)$$

$$B = \mu_0 (H + M)$$

$$M = \chi (H + M)$$

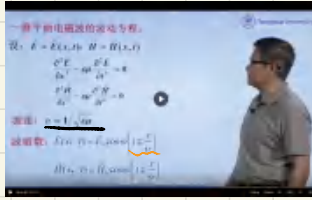


波动方程

$$\nabla^2 E - \epsilon \mu \frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 H - \epsilon \mu \frac{\partial^2 H}{\partial t^2} = 0$$

$$V^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$



一: 沿着x轴正方向
十: 沿着x轴负方向



电磁波: $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$

(1) 横波: $E \perp k, H \perp k$

(2) E, H 互相垂直且垂直于传播方向

(3) $H = \sqrt{\epsilon \mu} E$

真空中: $E = \sqrt{\frac{\mu_0}{\epsilon_0}} H = \sqrt{\frac{1}{\epsilon_0 \mu_0}} B = c B$

$B = \frac{E}{c}$

$E = c B$

坡印廷矢量

电磁场能量密度

$$W = \frac{1}{2} D \cdot E + \frac{1}{2} B \cdot H$$

各向同性: $W = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2$

$$W = \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2$$

$$= \frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu \frac{\epsilon}{\mu} E^2$$

$$= \epsilon E^2$$

$$= \epsilon E \sqrt{\frac{\mu}{\epsilon}} H$$

$$= \epsilon \sqrt{\frac{\mu}{\epsilon}} E H$$

$$= \sqrt{\mu \epsilon} E H$$

$$W = \frac{E H}{v} \rightarrow \text{波速}$$

电磁波能量密度 单位面积传播能量

$$S = \frac{W \cdot v \cdot \Delta t \cdot \Delta S}{\Delta t \cdot \Delta S} = W v = E H$$

$$\frac{J}{m^2} \cdot \frac{m}{s} = \frac{J}{m \cdot s}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

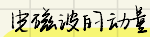
坡印廷矢量 $\Rightarrow$ 瞬时能量

电磁波强度: 能量密度在一个周期内平均值

$$I = \bar{S} = \frac{1}{2} \epsilon_0 H_0 \propto E_0^2 \text{ or } H_0^2$$

$\sim E_0^2$

E_0, H_0 : 峰值



能量密度 $w = \frac{\vec{E} \cdot \vec{H}}{v}$ $\vec{S} = \vec{E} \times \vec{H}$

质量密度 $m = \frac{W}{c^2} = \frac{2H}{c^2 v}$

动量密度 $\vec{g} = m\vec{v} = \frac{\vec{E} \times \vec{H}}{c^2} = \frac{\vec{S}}{c^2}$

$$g = \frac{E_H}{c^2} = \frac{E_H}{v \cdot c^2} \cdot v = \frac{E_H}{c^2}$$

能量密度 · 波速 = 能流密度



$$\frac{F}{S} = g^v = \frac{ZH}{C} \cdot C$$

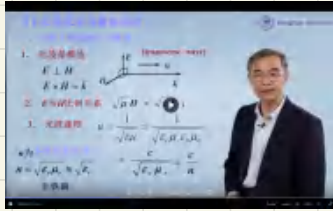
$$= \frac{ZH}{C}$$

$$p = \frac{E_H}{c} = \epsilon_0 \overline{E}^2$$

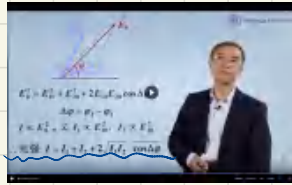
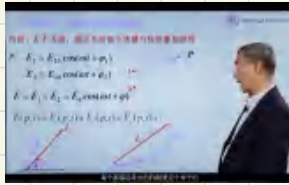
波动光学

光干涉→叠加时,某区域光强的分布

条件: 振动方向相同、频率相同、相位差恒定

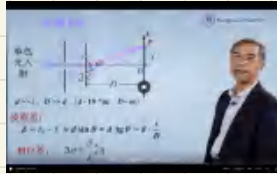
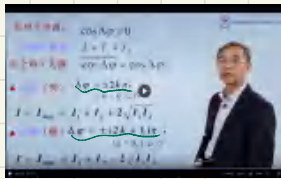


$$n = \sqrt{\epsilon_r \mu_r}$$



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

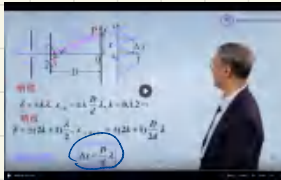
双缝干涉



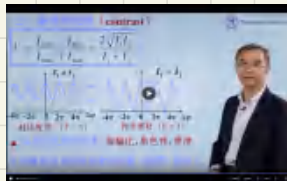
$$S = d \cdot \frac{x}{D}$$

$$x = D \tan \theta \approx D \sin \theta = \frac{D}{d} \delta x$$

$$\Delta x = \frac{D}{d} \lambda$$



$$\Delta x = \frac{D}{d} \lambda$$

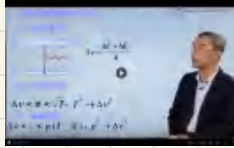


$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

双缝干涉光强分布: $I = 2I_0(1 + \cos \phi)$, $\phi = \frac{2\pi d \sin \theta}{\lambda}$



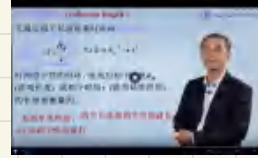
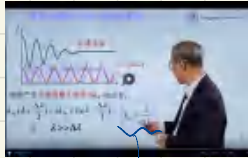
准单色光: 在某个中心波长附近有一定波长范围
谱线宽度



$$K(\lambda + \frac{\Delta\lambda}{2}) = (K+1)(\lambda - \frac{\Delta\lambda}{2})$$

$$\lambda \pm \frac{\Delta\lambda}{2} \sim \lambda \pm \frac{\Delta\lambda}{2}$$

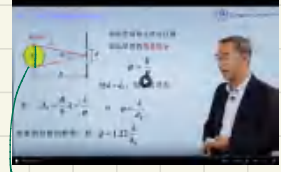
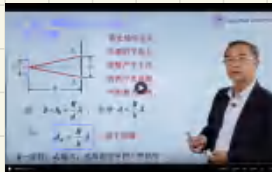
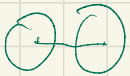
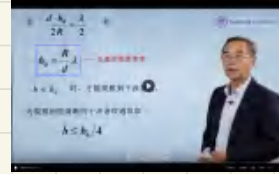
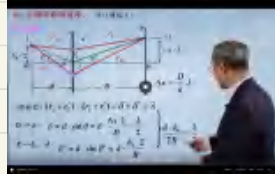
$$\delta M = \frac{\lambda^2}{\Delta\lambda} \quad K = \frac{\Delta\lambda}{\lambda}$$



$$K = \frac{\Delta\lambda}{\lambda}$$

$$b_0 = \frac{R}{\lambda} \lambda$$

计有介质
 $\Rightarrow \lambda \Rightarrow \frac{\lambda}{n}$
代替



$$d_0 = \frac{R}{\lambda} \lambda$$

$$d_0 = \frac{R}{\lambda} = \frac{R}{\lambda}$$

$$d_0 = \frac{R}{\lambda} \lambda$$

$$b = \frac{R}{\lambda} \lambda$$

$$d_0 = 2 \times \frac{d_0}{R} = \frac{d_0}{R}$$

$$= \frac{\lambda}{b}$$

拓展光源

$$d \leq \frac{R}{\lambda}$$

$$\psi = 2 \times \frac{\psi}{R}$$

$$= 2 \times \tan \frac{\psi}{R}$$

$$= 2 \times \frac{\psi}{R}$$

$$= \frac{\psi}{d_0}$$

$$= \frac{\lambda}{d_0}$$

光程

介质中:



$$\Delta\varphi = \varphi_b - \varphi_a = \frac{r}{\lambda} 2\pi$$

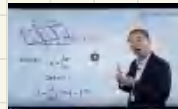
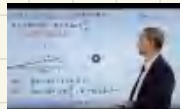
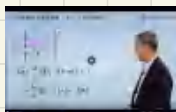
λ : 介质中波长

$$\lambda = \frac{v}{\nu} = \frac{c}{n\nu} = \frac{c}{\nu}$$

ν : 频率

$$\Delta\varphi = \frac{r}{\lambda} 2\pi = \frac{nr}{\lambda} 2\pi$$

nr : 真空中的与介质中路径 r 相应的路程



$m\lambda \rightarrow n\lambda$
半波损失: 波疏 $\rightarrow$ 波密

$$\begin{cases} \sin i = n \sin r \\ \delta = 2ne \cos r + \frac{\lambda}{2} \end{cases}$$

计 等厚 $\Rightarrow$ 垂直 $\Rightarrow \cos r = 1$
计 等倾 $\Rightarrow r, i$

等厚干涉

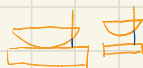
$$L = \frac{\lambda}{2n\theta}$$

$$\delta(e) = 2ne + \frac{\lambda}{2}$$

$$\begin{aligned} 2n\theta &= \lambda \\ L &= \frac{\lambda}{2ne n} \end{aligned}$$

$$\begin{aligned} \delta(e_1) &= 2ne_1 + \frac{\lambda}{2} \\ \delta(e_2) &= 2ne_2 + \frac{\lambda}{2} \\ \delta(e_1) - \delta(e_2) &= 2n(e_1 - e_2) \\ &= 2n\Delta e \\ &= \lambda \end{aligned}$$

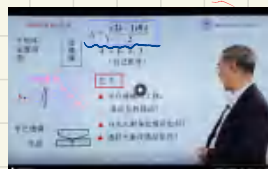
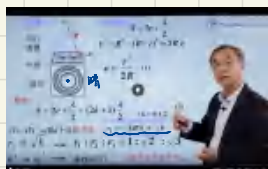
透射时, 干涉条纹四周向中心靠拢



$$\frac{2r^2}{2R} + \frac{\lambda}{2} = (2k+1)\frac{\lambda}{2}$$

$$\begin{aligned} \frac{r^2}{R} &= k\lambda \\ r &= \sqrt{Rk\lambda} \end{aligned}$$

牛顿环应用: $r_m^2 - r_k^2 = mR\lambda$



等倾干涉

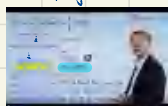
$$\begin{aligned} \sin i &= n \sin r \\ 2ne \cos r + \frac{\lambda}{2} \end{aligned}$$

小心全反射

$$\begin{aligned} 2ne \cos r + \frac{\lambda}{2} &= k\lambda \\ 2ne \cos r &= (k - \frac{1}{2})\lambda = \frac{2k-1}{2}\lambda \\ \cos r &= \frac{(2k-1)\lambda}{4ne} \end{aligned}$$

$$\sin i = n \sqrt{1 - \left(\frac{(2k-1)\lambda}{4ne} \right)^2} \leq 1$$

$\frac{\lambda}{2}$ 是倾角



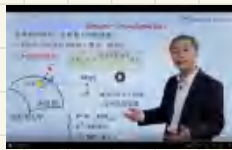
$$\Delta d = N \cdot \frac{\lambda}{2} \Rightarrow \Delta S = 2\Delta L$$

光的衍射

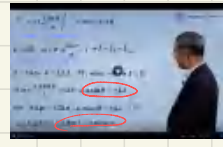
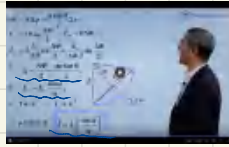
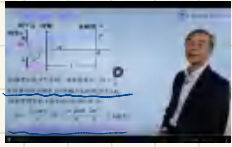
传播过程中经过障碍物边缘

$$\begin{aligned} \Delta s_1 &= k_1 \lambda \\ \Delta s_2 &= k_2 \lambda \\ \Delta(\Delta s) &= N \lambda = 2 \Delta(\Delta L) \\ \therefore \Delta d &= \frac{N \lambda}{2} \end{aligned}$$

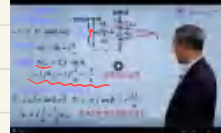
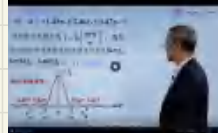
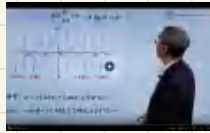




主极大 + 暗纹



次极大



$$\Delta x = \frac{1}{2} \Delta x_0 = \frac{1}{2} \Delta x_0 = \frac{1}{2} \Delta x_0$$



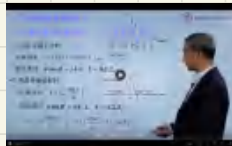
121:



光栅常数 $d = a + b$

光栅衍射

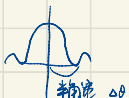
大量等宽等间距平行狭缝构成元件



主极大: $\sin \alpha \beta = \sin \theta = 0$

$N=2$ 次极大 $\sin \alpha \beta \neq 0$ $\sin \alpha \beta = 1$

$N=1$ 暗纹 $\sin \alpha \beta \neq 0$ $\sin \alpha \beta = 0$



$$I_p = I_0 \left(\frac{\sin \alpha \beta}{\alpha \beta} \right)^2 \left(\frac{\sin N \alpha \beta}{N \alpha \beta} \right)^2$$

单缝衍射因子

$$\beta = \frac{\pi d \sin \theta}{\lambda}$$

$$\alpha = \frac{\pi a \sin \theta}{\lambda}$$

$$\Delta \theta = \frac{\pi d \cos \theta}{\lambda} \Delta \theta$$

$$\Delta \beta = \frac{\pi d \cos \theta}{\lambda} \Delta \theta$$

$$\Delta \theta = \frac{\lambda \Delta \beta}{\pi d \cos \theta}$$

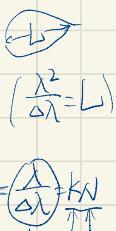
$$\Delta \beta = \frac{\pi}{N}$$

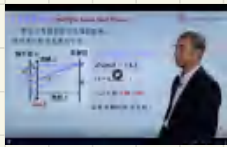
$$\Delta \theta = \frac{\lambda}{N d \cos \theta}$$

$$d \sin \theta = k \lambda$$

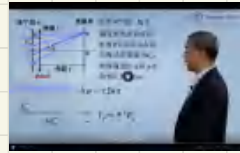
$$d \cos \theta \Delta \theta = k \lambda$$

$$\Delta \theta_2 = \frac{k \lambda}{d \cos \theta} = \Delta \theta_1 = \frac{\lambda}{N d \cos \theta}$$





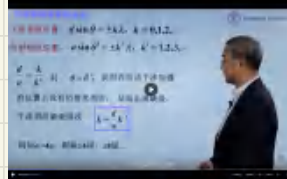
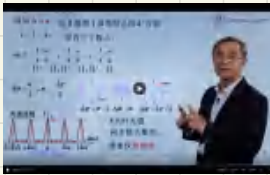
主极大 $d \sin \theta = \pm k \lambda$
 $\Delta \phi = \pm 2k\pi$
 $\vec{E}_p = N \vec{E}_1$ $I_p \propto N^2 E_p^2$



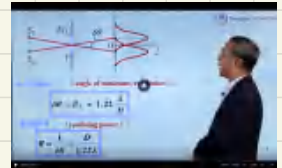
相邻主极大间有 $N-1$ 个暗纹和 $N-2$ 个次极大



$d \sin \theta = \pm \frac{k}{N} \lambda$

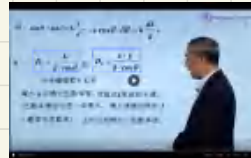
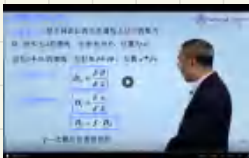


分辨率



透镜: $D \sin \theta = 1.22 \lambda$ $R = \frac{1}{50} = \frac{D}{1.22 \lambda}$

$\frac{\Delta \theta}{\lambda} \rightarrow R$



老栅色散: $D_0 = \frac{d\theta}{d\lambda} = \frac{k}{d \cos \theta}$
 $D_1 = \frac{d\theta}{d\lambda} = \frac{k f}{\lambda \cos \theta}$

老栅分辨: $R = \frac{\lambda}{\Delta \lambda} = N k$

例 干涉

$$I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin \beta}{\sin \beta} \right)^2$$

$$\alpha = \frac{\pi a \sin \theta}{\lambda}$$

$$\beta = \frac{\pi d \sin \theta}{\lambda}$$

① 干涉

主极大 $\Rightarrow \begin{cases} \sin \alpha = 0 \\ \sin \beta = 0 \end{cases}$

暗纹 $\Rightarrow \begin{cases} \sin \alpha \neq 0 \\ \sin \beta = 0 \end{cases}$

次极大 $\Rightarrow \begin{cases} \sin \alpha \neq 0 \\ \sin \beta \neq 0 \end{cases}$

$\beta = k\pi = \frac{\pi d \sin \theta}{\lambda}$

$d \sin \theta = \lambda k$

$\frac{k\pi}{N} = \frac{\pi d \sin \theta}{\lambda}$

$d \sin \theta = \frac{k\lambda}{N}$ $k=1, 2, \dots, N-1$

$\alpha \sin \theta = (2k+1) \frac{\lambda}{2}$

$a \sin \theta = 2k \frac{\lambda}{2}$

② 衍射

主极大

暗纹

次极大



角宽: 暗纹位置



$$2f \tan \theta = 2f \frac{\lambda}{a}$$

$$a \sin \theta = \lambda$$

副极大: 与衍射暗纹重合

$$d \sin \theta = k\lambda$$

$$a \sin \theta = k_2 \lambda$$

$$\frac{d}{a} = \frac{k_2}{k_1}$$

主极大之间
N-1个次极大
N-1个暗纹

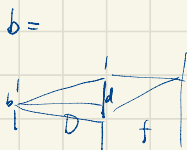
① 干涉中央暗纹

$$\frac{\pi d \sin \theta}{\lambda} = \beta = \frac{k\pi}{N}, k=1, \dots, N-1$$

(中央)

$$d \sin \theta = \frac{\lambda}{N} \Rightarrow d \theta_1 = \frac{\lambda}{N}$$

2\theta_1 中央宽度



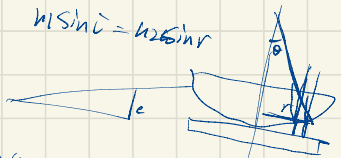
② 干涉环状角位置 (看例题)

$$\beta = \frac{\pi d \sin \theta}{\lambda} = \frac{k\pi}{N}, k=1, \dots, N-1$$

$$\Delta \beta = \frac{\pi d \cos \theta \Delta \theta}{\lambda} = \frac{\pi}{N}$$

$$d \cos \theta \Delta \theta = \frac{\lambda}{N}$$

$$\begin{cases} 2ne \cos r = \delta \\ n_1 \sin i = n_2 \sin r \end{cases}$$



$$R \sin \theta = r \quad \theta = \frac{\pi}{2}$$

$$e = R \frac{\theta^2}{2}$$

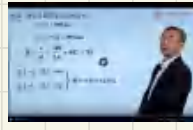
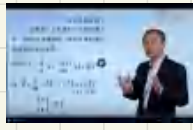
$$e = \frac{R}{2} \times \frac{r^2}{R^2}$$

$$1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}$$

$$= 2 \left(\frac{\theta}{2} \right)^2$$

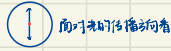
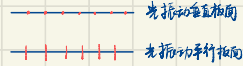
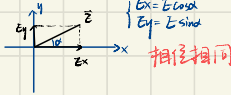
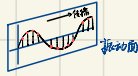
$$= \frac{\theta^2}{2}$$

解: 中央极大

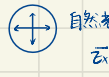


光偏振状态

一. 线偏振光



二. 自然光 (非偏振光)

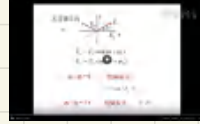
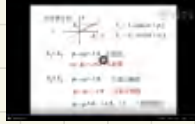
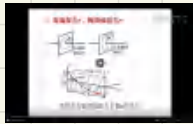
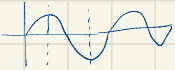


自然光的分解
 $E_z = E_y$

相位不同



自然光



偏振度

$$P = \frac{I_p}{I_n + I_p}$$

I_n : 部分偏振光中包含的自然光的强度
 I_p : 部分偏振光中包含的完全偏振光的强度

完全偏振光 (线、圆、椭圆) $P=1$

自然光 (非偏) $P=0$

部分偏振光 $0 < P < 1$

过偏和检偏 $\rightarrow$ 时有消光 $\Rightarrow$ 线偏振光
自然光 $\Rightarrow$ 圆偏振光 $\Rightarrow$ 总有半光透过去



$$I = I_0 \cos^2 \alpha$$

偏振面: $\updownarrow$ 可透

布儒斯特角

$$\begin{aligned} n_1 \sin i_0 &= n_2 \sin r_0 \\ &= n_2 \cos i_0 \\ \tan i_0 &= \frac{n_2}{n_1} \end{aligned}$$



反射、折射光的偏振

自然光斜入射: $\left\{ \begin{array}{l} P \text{ 波: 偏振面平行于入射面} \\ S \text{ 波: } \perp \end{array} \right. > \text{反射率不同}$

反射光是完全线偏振光, 只有上分量

折射光是部分线偏振光

$$\begin{aligned} n_1 \sin i_0 &= n_2 \sin(\frac{\pi}{2} - i_0) = n_2 \cos i_0 \\ \tan i_0 &= \frac{n_2}{n_1} \end{aligned}$$

散射光的偏振

双折射



O光: $n_1 \sin i_1 = n_2 \sin r_1$

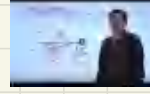
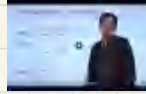
光轴沿光轴传播 不发生双折射 $\rightarrow$ 沿光轴
not a 轴

主平面: 光线 and 光轴组成的平面

No.



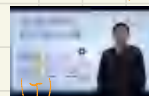
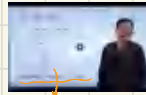
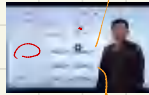
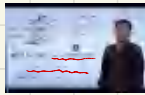
$\therefore v_e > v_o$ 传播速度
e光沿光轴方向



e光: 以光轴为对称轴的旋转椭球面



沿传播方向, 相位差 $\frac{\pi}{2} \rightarrow$ 圆

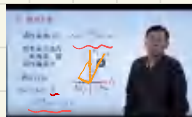


Σ 波片带来 π 相位差

沿光轴 $\rightarrow$ 沿光轴
max/min 光轴

偏振光干涉

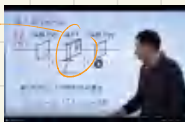
O光: $\perp$ 光轴; e光: 沿光轴
光线: 传播方向 $\perp$ 纸面 $\perp$ P1



$+\pi$: 因为 A_{2o}, A_{2e} 同差 π .



光轴方向
平行波片表面



$I = A_{2o}^2 + A_{2e}^2 + 2A_{2o}A_{2e}\cos\alpha$

$A_{2o} = A_{2e} = A_1 \sin\alpha \cos\alpha$

应力: $|n_e - n_o| = k \frac{F}{S}$, $|n_e - n_o| = \frac{2\pi d}{\lambda} |n_e - n_o| + \pi = \frac{k d 2\pi}{\lambda} \cdot \frac{F}{S} + \pi$
双折射

旋光现象

磁致旋光: $\psi = VdB$
 费德尔定律

方向与磁场方向有关 $\Rightarrow$ 光线沿 B 方向 B 偏振面旋转方向相反



量子物理

热辐射平衡 物体辐射的能量等于同一时间吸收的能量

黑色辐射本领更强

黑体: M_ν 最大且只与温度有关, 与材料及表面状态无关
 辐射本领

吸收, 对外辐射电磁波, 但不反射.

辐射比普朗克量子论 $m \ll \frac{h\nu}{c}$ 的 ν

(连续)

辐射

黑体辐射

$$V_m = C_\nu T$$

$$\lambda \lambda = \sigma T^4 = n h \nu$$

$\rightarrow$ ν 以 K 为单位

分析

光电效应

截止电压: 与入射光强度无关

光电流 $\sim \nu$
 $\sim V$

饱和光电流强度 i_m 与入射光强 I 成正比 (V 足够)

$$\frac{1}{2} m v_m^2 = e U_c = e (K\nu - U_0) = h e (\nu - \nu_0)$$

爱因斯坦相对论

$$E = mc^2 = h\nu \rightarrow m = \frac{h\nu}{c^2} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad m_0 = 0$$

光静止质量为 0

$$E^2 = p^2 c^2 + m_0^2 c^4 \rightarrow p = \frac{E}{c} = \frac{h\nu}{c}$$

$$\vec{p} = \frac{E}{c} \vec{n} = \frac{h\nu}{c} \vec{n}$$

光 $\rightarrow$

$$h = \frac{h}{2\pi}$$

光能量 $E = h\nu$
 $= pc$
 $= mc^2$

$$E = \hbar \omega \quad \vec{p} = \hbar \vec{k}$$

$$\text{波函数 } \psi(\vec{r}, t) = A e^{i(\vec{p} \cdot \vec{r} - Et)}$$

$|\text{波函数}|^2 = \text{概率密度} \sim \text{光子密度}$

$|\psi(\vec{r}, t)|^2 dV \Rightarrow \text{概率} \quad \text{这时光子出现的概率}$

概率密度

康普顿

1. X 射线入射到原子, 散射波与入射波入相同

2.

自由电子, 能量动量守恒

电子吸收光子, 放出光子

$$E = h\nu = \hbar \omega$$

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

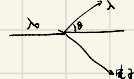
$$\Delta \lambda = \lambda - \lambda_0 = \frac{h}{m_0 c} (1 - \cos \theta)$$

m_0 电子静质量

$$h\nu + m_0 c^2 = h\nu' + m_0 c^2 + E_k$$

能量守恒

$$h\nu = h\nu' + E_k$$



光子碰撞

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = p_0 + p_{\text{电}}$$

$$h: 6.63 \times 10^{-34}$$

$$MeV: 10^6 \times 1.6 \times 10^{-19}$$

物质波

$$\frac{dW}{dx} = v \quad \frac{W}{L} = \frac{c^2}{v} \quad \text{粒子运动速度是物质波的群速度}$$

物质低速 $\Rightarrow$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

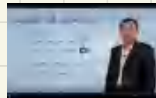
波函数: 只具有统计意义

$$\int_{\Omega} |\psi(r,t)|^2 dv = 1 \quad \text{归一化} \quad \psi(r,t) = \psi_0 e^{-\frac{i}{\hbar}(Et - p \cdot r)}$$

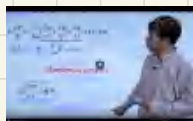
1. 全空间

$$\text{波归一化因子 } C \quad \int_{\Omega} |\psi|^2 dv = 1$$

状态叠加: Δ 是对波函数数量加, 再算概率



波函数寿命与能量宽度成反比



$$\text{定态: } \psi(r,t) = \psi(r) \cdot A_0 e^{-\frac{i}{\hbar}Et}$$

如果粒子初始处于排定态, 则它始终处于排定态。

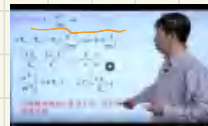
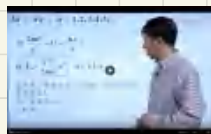
① 自由电子: $U(x) = 0$ 处处

② (默认): $(0,a)$ 内 $U(x) = 0$

取势阱底作为势能零点

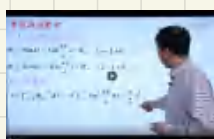
有限'和无限比:

有限基态能量比无限低



③ 一维谐振子

$$U(x) = \frac{1}{2}kx^2$$



$$a: \text{宽度}$$

$$A = \sqrt{\frac{2}{a}}$$

$$E_n = \frac{\hbar^2 k^2 \pi^2}{2ma^2} = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) e^{-i\omega_n t} \quad \text{[特解]}$$

$$n=1, 2, \dots \quad \omega_n = \frac{E_n}{\hbar}$$

注意

but 通解可以叠加

一维谐振子

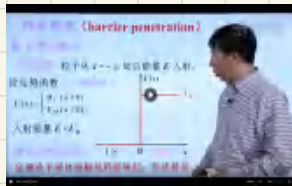
1. 动能 $U(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$, $\omega = \sqrt{\frac{k}{m}}$ 这里的k不是那个k

2. 能量 $E_n = (n + \frac{1}{2})\hbar\omega = (n + \frac{1}{2})\hbar\nu$, $n = 0, 1, \dots$
 $E_0 = \frac{1}{2}\hbar\nu$, $E_1 = \frac{3}{2}\hbar\nu, \dots$

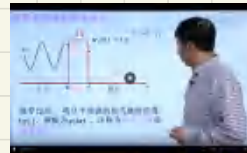
$\Delta E = \hbar\nu$

有选择定则: 能级跃迁满足 $\Delta n = \pm 1$

势垒穿透



量子: 粒子可透入
 $|\psi(x)|^2 \propto e^{-\frac{2\sqrt{2m(U_0-E)}}{\hbar}x}$
 $U_0 \uparrow, x \uparrow \Rightarrow$ 透入概率 $\downarrow$



不确定性

薛定谔方程

1D薛定谔方程: $[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)]\psi(x,t) = E\psi(x,t)$

△ 已知求E可用分离

△ 求解方程求 $\psi(x,t)$ 可试探分离 $\psi(x,t) = \psi(x)\phi(t)$

△ 已知解 $\Rightarrow$ 可波 $\psi(x,t) = A[f_1\omega_1\phi_1 + f_2\omega_2\phi_2]$ 并代入

关于 $U(x)$:

- ① 自由粒子: $U(x) = 0$ 处处
- ② (默认): $(0, a)$ 内 $U(x) = 0$
- ③ 一维谐振子
 $U(x) = \frac{1}{2}kx^2$

△ 公式:

$E = \frac{p^2}{2m} = \hbar\nu = \hbar\omega$

$p = \frac{h}{\lambda} = \hbar \frac{2\pi}{\lambda} = \hbar k$

$\therefore \frac{\hbar^2 k^2}{2m} = \hbar\omega \therefore \omega = \frac{\hbar k^2}{2m}$

对无限 $k = \frac{2\pi}{\lambda} = \frac{\pi x}{a}$

原子中的电子

$$n = 1, 2, 3, \dots$$

$$\text{波数 } \tilde{\nu} = R \left(\frac{1}{n^2} - \frac{1}{n'^2} \right), \quad n' = n+1, n+2, n+3, \dots$$

$$= \frac{1}{\lambda}$$

$$R = \frac{m_e e^4 / h^2}{2(4\pi\epsilon_0)^2 \lambda^2}$$

$$\Delta m v n r = n \hbar$$

$$\text{光: } p = \frac{h\nu}{c} = \frac{mc}{\lambda}$$

量子化条件: $\oint p_{\phi} dq_{\phi} = n h$

$$r n = n^2 r_1 = n^2 \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2}$$

折合质量 $m = \frac{m_1 m_2}{m_1 + m_2}$

$$E_n = \frac{1}{2} m_e v^2 = - \frac{m_e e^4}{2(4\pi\epsilon_0)^2 \hbar^2} \cdot \frac{1}{n^2}$$

玻尔模型: $E_1 \left(\frac{1}{n^2} - \frac{1}{n'^2} \right) = h\nu = \frac{1}{2} m_e v^2$

原子的能量守恒: $E_1 = m_e v^2$ (原子的静止)

$$E_1 \left(\frac{1}{n^2} - \frac{1}{n'^2} \right) = h\nu = \frac{1}{2} m_e v^2$$

无外B

m_l 无影响

有外B

$\Rightarrow$ 能级分裂

$$L_z = m_l \hbar$$

$$i > j, E_i > E_j$$

$$L = \sqrt{l(l+1)} \hbar$$

$$L_z = m_l \hbar$$

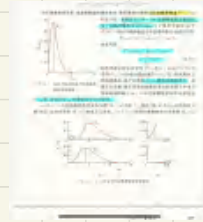
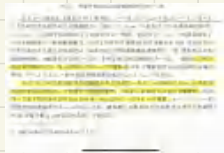
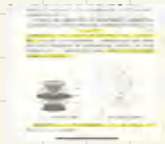
$$L_z \text{ 特征值是 } m_l \hbar, \quad m_l = 0, \pm 1, \pm 2, \dots$$

$$| \text{波函数} \rangle = \phi_m(\rho) = A e^{i m_l \phi}$$

m_l 磁量子数

n 主量子数

l 轨道量子数



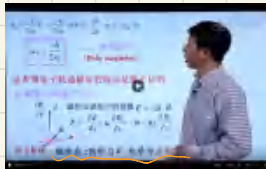
$l=0$ 除开
 $E_{n,l}$ 分为 2
能级

电子自旋与自旋轨道耦合

磁矩与角动量 $\vec{\mu} = -\frac{e}{2m_e} \vec{L}$

$$\mu_z = -\frac{e}{2m_e} L_z = -\frac{e}{2m_e} m_l \hbar = -\frac{e \hbar}{2m_e} m_l = -\mu_B m_l$$

$$\mu_B = \frac{e \hbar}{2m_e} \text{ 玻尔磁子}$$



与不同 m_l

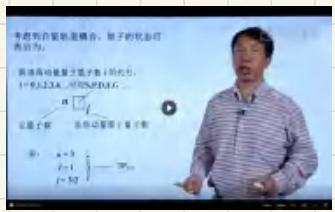
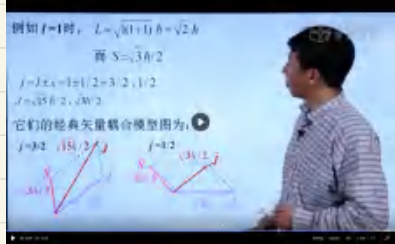


$$S_z = m_s \hbar$$

$$S = \sqrt{s(s+1)} \hbar$$

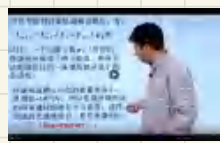


$$S = \frac{1}{2} \hbar$$



$\vec{j} = \vec{l} + \vec{s}$
 $\vec{j} = \vec{L} + \vec{S}$
 矢量合成

磁量子数: $n \rightarrow l, m$, 靠近核概率 $\uparrow$, $E_p \downarrow$
 $E_{nl} = \frac{-13.6 \text{ eV}}{(n - a_{nl})^2}$ a_{nl} : 量子数亏损



泡利不相容原理

全同粒子



1. 费米子: $S = \text{半整数}$ (自旋电子) $\psi_{(1,2)} = -\psi_{(2,1)}$

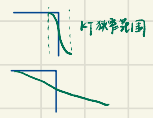
$$= \frac{1}{\sqrt{2}} [\psi_A(1)\psi_B(2) - \psi_A(2)\psi_B(1)]$$

不可能有2个全同的费米子处于同一的量子态

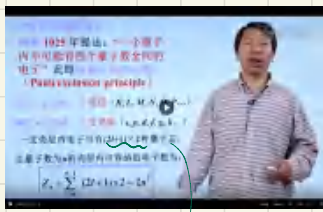
2. 玻色子: $S = 0 \text{ or 整数}$ $\psi_{(1,2)} = \psi_{(2,1)}$



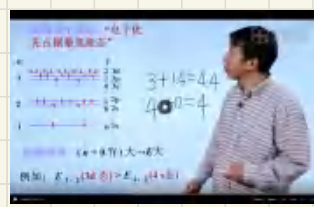
费米初速 $v_F = \sqrt{2mE_F}$
 费米速度 $v_F = \sqrt{2E_F/m}$
 $n = n(T)$
 $n(0) = E_F$ 费米能量
 $n(T) \sim E_F$
 费米温度 $T_F = E_F/k$



$T=0$ 集中占据态
 $n(0)$



x_2 : 自旋向上/向下

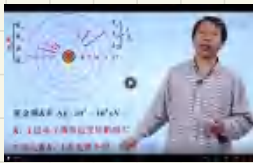


$n+0.7l$

X射线

X射线谱 { 连续谱: 轫致辐射
线状谱: 内层电子跃迁

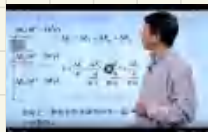
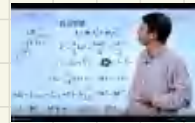
轫致辐射: 辐射强度 $I \propto \alpha^2 \propto \frac{(Ze)^4}{m^2}$ Z : 辐射元素的原子序数
 $E = eU = h\nu_{\min} = \frac{hc}{\lambda_{\min}} \therefore \lambda_{\min} = \frac{hc}{eU} \propto \frac{1}{U} \propto \frac{1}{V}$ 和靶元素无关



莫塞莱定律: $\sqrt{\nu_{K\alpha}} = 0.496 \times 10^8 (Z-1)$
 $\hookrightarrow$ 确定 Z

分子光谱

产生: $\left. \begin{array}{l} \text{电子运动 } E_e \\ \text{分子振动 } E_v \\ \text{分子转动 } E_r \end{array} \right\} E = E_e + E_v + E_r$

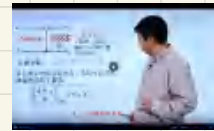


激光

1. 自发辐射 与光强无关



2. 吸收 正比于光的强度



光子数分布反转

要产生激光，必须满足：

$$\left(\frac{dN_2}{dt} \right)_{\text{spont}} > \left(\frac{dN_2}{dt} \right)_{\text{spont}}$$

$$N_2 > N_1 \rightarrow \text{非热平衡}$$

激活物质：

1. 有三级以上能级系统
2. 上能级为亚稳态 (自发辐射系数小) $\Rightarrow$ 呆在那里，光子数大
3. 下能级不是基态，且对下能级自发辐射大 $\Rightarrow$ 光子数少

光学谐振腔

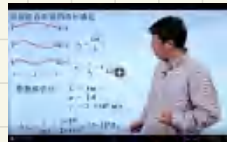


作用：

1. 方向性
2. 光放大
3. 单色性

形成纵模和横模

1) 纵模



1. 折射率

2) 横模

